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One-loop results for the quark-gluon vertex in arbitrary dimension*

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Results on the one-loop quark-gluon vertex with massive quarks are reviewed, in an arbitrary covariant gauge and in arbitrary space-time dimension. We show how it is possible to get on-shell results from the general off-shell expressions. The corresponding Ward-Slavnov-Taylor identity is discussed.

1. INTRODUCTION

The quark-gluon vertex is one of the fundamental objects in Quantum Chromodynamics (QCD) [1]. To calculate various higher-order QCD effects we should have a detailed knowledge of this vertex. For the one-loop quark-gluon vertex we have two contributions. The "Abelian" contribution is equal, up to an overall factor, to the fermion-photon vertex in QED. In the four-dimensional case, off-shell results for the latter have been obtained in [2] (Feynman gauge) and [3] (an arbitrary covariant gauge). In three dimensions, results for the massless case are available in [4].

In special limits some results are also available for the second, non-Abelian contribution. In Ref. [5] (see also [6]), the "symmetric" case, $p_1^2 = p_2^2 = p_3^2$, was treated in an arbitrary covariant gauge, for massless quarks. For massive quarks, this $p_1^2 = p_2^2 = p_3^2$ case was examined in [7] (only the coefficient of γ_μ was calculated). Some configurations when the gluon (or quark) momentum vanishes were considered in [8]. Some on-shell results are available for massless quarks [9]. Recently we completed a study of the one-loop quark-gluon vertex in an arbitrary covariant gauge and space-time dimension [10].

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2. FORMALISM

The lowest-order quark-gluon vertex is

$$g(T^a)_{ji}\,[\gamma_\mu]_{\beta\alpha}\,, \tag{1}$$

where T^a are colour matrices corresponding to the fundamental representation of the gauge group. We will consider the SU(N) group, with N the number of colours. Extracting the overall colour structure, we can present the "dressed" (one-particle irreducible) quark-gluon vertex as

$$\Gamma^a_\mu(p_1,p_2,p_3) = gT^a\,\Gamma_\mu(p_1,p_2,p_3), \tag{2}$$

where p_1, p_2 are momenta of the quarks, p_3 is the gluon momentum, all of which are ingoing, $p_1+p_2+p_3=0$.

At the one-loop level, we have two contributions to the quark-gluon vertex (see Fig. 1). Their colour factors are proportional to $\left(C_F - \frac{1}{2}C_A\right)$ and C_A, where C_F and C_A denote eigenvalues of the quadratic Casimir operator in the fundamental and adjoint representations, respectively. For the SU(N) gauge group, $C_A = N$, and $C_F = (N^2-1)/(2N)$.

2.1. Ward-Slavnov-Taylor (WST) identity

The WST identity [11] for the quark-gluon vertex $\Gamma_\mu(p_1,p_2,p_3)$ reads (see, e.g., in [12])

$$p_3^\mu \Gamma_\mu(p_1,p_2,p_3) = G(p_3^2)\left[S^{-1}(-p_1)\;H(p_1,p_2,p_3) - \overline{H}(p_2,p_1,p_3)\;S^{-1}(p_2)\right], \tag{3}$$

where $G(p^2)$ is the scalar function associated with the ghost propagator (the latter is proportional to

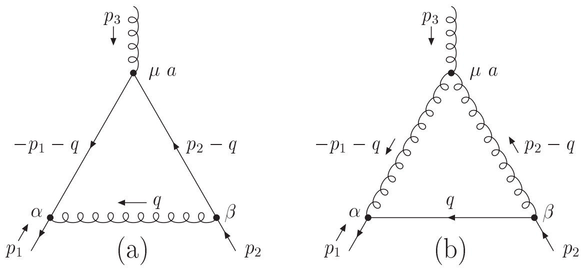

Figure 1. The two one-loop diagrams.

$G(p^2)/p^2$); S^{-1} is the inverse quark propagator; the function H (and the "conjugated" function $\overline{H}$) involves the complete four-point quark-quark-ghost-ghost vertex (for details, see [12,10]).

At the one-loop level, it is convenient to "split" the WST identity into two separate relations, corresponding to the contributions of the two diagrams. To do this, we need to rewrite the one-loop contribution to the r.h.s. of (3) in terms of colour coefficients $(C_F - \frac{1}{2}C_A)$ and C_A, by analogy with the two contributions to the l.h.s. On the r.h.s., all one-loop contributions are proportional to C_A, except for the quark self energies, which contain C_F. Therefore, all we need to do is to represent this C_F as $(C_F - \frac{1}{2}C_A) + \frac{1}{2}C_A$. In this way, we get two separate WST identities for the contributions of diagrams a and b in Fig. 1[3],

$$
\begin{aligned}
&p_3^\mu \Gamma_\mu^{(1a)}(p_1,p_2,p_3) = \left(C_F - \tfrac{1}{2}C_A\right)\ C_F^{-1} \\
&\quad\times \left[S^{-1}(-p_1) - S^{-1}(p_2)\right]^{(1)}, \qquad (4)
\end{aligned}
$$

$$
\begin{aligned}
&p_3^\mu \Gamma_\mu^{(1b)}(p_1,p_2,p_3) = \left[S^{-1}(-p_1)\right]^{(0)} H^{(1)}(p_1,p_2,p_3) \\
&-\overline{H}^{(1)}(p_2,p_1,p_3)\ \left[S^{-1}(p_2)\right]^{(0)} \\
&+\tfrac{1}{2}C_A\ C_F^{-1}\ \left[S^{-1}(-p_1) - S^{-1}(p_2)\right]^{(1)}\ H^{(0)} \\
&+2G^{(1)}(p_3^2)\ \left[S^{-1}(-p_1) - S^{-1}(p_2)\right]^{(0)}\ H^{(0)}. \qquad (5)
\end{aligned}
$$

The first identity has (up to a factor) the same structure as the Abelian (QED) identity. The second identity is a "non-Abelian" one.

2.2. Decomposition of the vertex

Because of the WST identity (3), it is convenient to decompose the quark-gluon vertex into a longitudinal part and a transverse part, as proposed in [2],

$$\Gamma_\mu(p_1,p_2,p_3) = \Gamma_\mu^{(\mathrm{L})}(p_1,p_2,p_3) + \Gamma_\mu^{(\mathrm{T})}(p_1,p_2,p_3) \qquad (6)$$

where

$$\Gamma_\mu^{(\mathrm{L})}(p_1,p_2,p_3) = \sum_{i=1}^{4} \lambda_i(p_1^2,p_2^2,p_3^2)\, L_{i,\mu}(p_1,p_2), \qquad (7)$$

$$\Gamma_\mu^{(\mathrm{T})}(p_1,p_2,p_3) = \sum_{i=1}^{8} \tau_i(p_1^2,p_2^2,p_3^2) T_{i,\mu}(p_1,p_2), \qquad (8)$$

with

$$
\begin{aligned}
L_{1,\mu} &= \gamma_\mu, \\
L_{2,\mu} &= (\not{p}_1 - \not{p}_2)(p_1 - p_2)_\mu, \\
L_{3,\mu} &= (p_1 - p_2)_\mu, \\
L_{4,\mu} &= \sigma_{\mu\nu}\,(p_1 - p_2)^\nu, \qquad (9) \\
T_{1,\mu} &= p_{1\mu}(p_2 p_3) - p_{2\mu}(p_1 p_3), \\
T_{2,\mu} &= \left[-p_{1\mu}(p_2 p_3) + p_{2\mu}(p_1 p_3)\right](\not{p}_1 - \not{p}_2), \\
T_{3,\mu} &= p_3^2 \gamma_\mu - p_{3\mu}\not{p}_3, \\
T_{4,\mu} &= \left[p_{1\mu}(p_2 p_3) - p_{2\mu}(p_1 p_3)\right]\sigma_{\nu\lambda}\, p_1^\nu p_2^\lambda, \\
T_{5,\mu} &= \sigma_{\mu\nu}\, p_3^\nu, \\
T_{6,\mu} &= \gamma_\mu(p_1^2 - p_2^2) + (p_1 - p_2)_\mu \not{p}_3, \\
T_{7,\mu} &= \tfrac{1}{2}(p_2^2 - p_1^2)\left[\gamma_\mu(\not{p}_1 - \not{p}_2) - (p_1 - p_2)_\mu\right] \\
&\quad -(p_1 - p_2)_\mu \sigma_{\nu\lambda}\, p_1^\nu p_2^\lambda, \\
T_{8,\mu} &= -\gamma_\mu \sigma_{\nu\lambda}\, p_1^\nu p_2^\lambda + p_{1\mu}\not{p}_2 - p_{2\mu}\not{p}_1. \qquad (10)
\end{aligned}
$$

The transverse part $\Gamma_\mu^{(\mathrm{T})}$ does not contribute to the WST identity (3), $p_3^\mu \Gamma_\mu^{(\mathrm{T})}(p_1,p_2,p_3) = 0$.

Furthermore, in Ref. [3] a modification of the basis (8)–(10) has been proposed, which has an advantage in dealing with kinematical singularities[4]. Namely, the transverse part is represented as

$$\Gamma_\mu^{(\mathrm{T})}(p_1,p_2,p_3) = \sum_{i=1}^{8} \widetilde{\tau}_i(p_1^2,p_2^2,p_3^2)\widetilde{T}_{i,\mu}(p_1,p_2), \qquad (11)$$

where

$$\widetilde{T}_{4,\mu} = \frac{2}{p_2^2 - p_1^2}\left[2T_{4,\mu} - p_3^2 T_{7,\mu}\right], \qquad (12)$$

$$\widetilde{\tau}_4 = \tfrac{1}{4}(p_2^2 - p_1^2)\tau_4, \qquad \widetilde{\tau}_7 = \tau_7 + \tfrac{1}{2}p_3^2\tau_4, \qquad (13)$$

[3] For a quantity X, we denote the zero-loop-order contribution as $X^{(0)}$, and the one-loop-order contribution as $X^{(1)}$, so that the perturbative expansion looks like $X = X^{(0)} + X^{(1)} + \ldots$.

[4] In Ref. [3], the notation σ_i and S_i was used for what we call $\widetilde{\tau}_i$ and $\widetilde{T}_i$.

whereas $\widetilde{T}_{i,\mu} = T_{i,\mu}$ $(i \neq 4)$ and $\widetilde{\tau}_i = \tau_i$ $(i = 1,2,3,5,6,8)$. Moreover, in the on-shell limit the following modifications of λ_2 and λ_3 turn out to be useful:

$$\widetilde{\lambda}_2 \equiv \lambda_2 + \tfrac{1}{2} p_3^2 \tau_2, \qquad \widetilde{\lambda}_3 \equiv \lambda_3 - \tfrac{1}{2} p_3^2 \tau_1 \ . \qquad (14)$$

Applying charge conjugation to the quark-gluon vertex, i.e., interchanging quark and antiquark, one obtains (see, e.g., in Ref. [3]):

$$\mathrm{C}\,\Gamma_\mu(p_1,p_2,p_3)\,\mathrm{C}^{-1} = -\Gamma_\mu^{\mathrm{T}}(p_2,p_1,p_3). \qquad (15)$$

Interchanging the quark momenta $(p_1 \leftrightarrow p_2)$ and using the fact that $\mathrm{C}\ \gamma_\mu\ \mathrm{C}^{-1} = -\gamma_\mu^{\mathrm{T}}$, one finds that all L_μ and T_μ are odd, except for $L_{4,\mu}$ and $T_{6,\mu}$, which are even. Thus, to satisfy Eq. (15) all λ's and τ's must be symmetric under the interchange of p_1^2 and p_2^2, except λ_4 and τ_6, which are odd. An important corollary is that in the case $p_1^2 = p_2^2 \equiv p^2$ the λ_4 and τ_6 functions must vanish,

$$\lambda_4(p^2,p^2,p_3^2) = 0, \qquad \tau_6(p^2,p^2,p_3^2) = 0\ . \qquad (16)$$

2.3. Integrals

The two one-loop quark-gluon vertex diagrams involve two triangle integrals:

$$J_2(\nu_1,\nu_2,\nu_3) \equiv \int \frac{\mathrm{d}^n q}{[(p_2-q)^2-m^2]^{\nu_1}[(p_1+q)^2-m^2]^{\nu_2}(q^2)^{\nu_3}}, \qquad (17)$$

$$J_1(\nu_1,\nu_2,\nu_3) \equiv \int \frac{\mathrm{d}^n q}{[(p_2-q)^2]^{\nu_1}\,[(p_1+q)^2]^{\nu_2}\,[q^2-m^2]^{\nu_3}}, \qquad (18)$$

where $n = 4-2\varepsilon$ is the space-time dimension in the framework of dimensional regularization [14]. The subscript of J indicates the number of massive propagators (see, e.g., in [13]).

Tensor integrals can be reduced to the scalar ones using the standard techniques [15] (see also in [16,17]). Using the integration-by-parts technique [18], all integrals with higher integer powers of propagators can be algebraically reduced to integrals with the powers equal to one or zero (for details, see [19]).

Two non-trivial integrals appear[5],

$$J_i(1,1,1) = \mathrm{i}\ \pi^{n/2}\ \eta\ \varphi_i(p_1^2,p_2^2,p_3^2;m), \qquad (19)$$

[5] We have extended the notation used in [20] by introducing the functions φ_i $(i=1,2)$, where i shows the number of massive propagators involved.

where we have extracted a factor

$$\eta \equiv \frac{\Gamma^2(\frac{n}{2}-1)}{\Gamma(n-3)}\Gamma(3-\tfrac{n}{2}) = \frac{\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\Gamma(1+\varepsilon). \qquad (20)$$

Then, the following two-point functions appear:

$$\begin{aligned} J_1(1,1,0) &= J_0(1,1,0) = \mathrm{i}\ \pi^{n/2}\ \eta\ \kappa_{0,3}\ , \\ J_1(0,1,1) &= J_2(0,1,1) = \mathrm{i}\ \pi^{n/2}\ \eta\ \kappa_{1,1}\ , \\ J_1(1,0,1) &= J_2(1,0,1) = \mathrm{i}\ \pi^{n/2}\ \eta\ \kappa_{1,2}\ . \end{aligned} \qquad (21)$$

We have introduced the functions $\kappa_i(p_l^2;m) \equiv \kappa_{i,l}$, where p_l $(l=1,2,3)$ is the external momentum of the two-point function, whereas the subscript $i=0,1,2$ shows how many of the two internal propagators are massive. For massive lines we can also get "tadpole" integrals:

$$\begin{aligned} &J_1(0,0,1) = J_2(1,0,0) = J_2(0,1,0) \\ &\quad = \mathrm{i}\ \pi^{n/2}\frac{\Gamma(1+\varepsilon)}{\varepsilon(1-\varepsilon)}(m^2)^{1-\varepsilon} = \mathrm{i}\ \pi^{n/2}\ \eta\ m^2\ \widetilde{\kappa}, \end{aligned} \qquad (22)$$

with

$$\widetilde{\kappa} \equiv \widetilde{\kappa}(m^2) \equiv \frac{\Gamma(1-2\varepsilon)}{\Gamma^2(1-\varepsilon)}\ \frac{1}{\varepsilon(1-\varepsilon)}\ (m^2)^{-\varepsilon}. \qquad (23)$$

Massless tadpoles vanish in the framework of dimensional regularization [14]. In four dimensions, the integrals (19) can be evaluated in terms of dilogarithms [21].

3. OFF-SHELL RESULTS

Before presenting selected results for the λ and τ functions, let us introduce the following notation for the occurring Gram determinants:

$$\begin{aligned} \mathcal{K} &\equiv p_1^2 p_2^2 - (p_1 p_2)^2\ , & (24)\\ \mathcal{M}_1 &\equiv \mathcal{K} + \left((p_1p_2)+m^2\right)^2\ , & (25)\\ \mathcal{M}_2 &\equiv (p_1^2-m^2)(p_2^2-m^2)p_3^2 + m^2(p_1^2-p_2^2)^2\ . & (26) \end{aligned}$$

We consider the contributions of diagrams a and b (see Fig. 1) separately,

$$\lambda_i^{(1)} = \lambda_i^{(1a)} + \lambda_i^{(1b)}\ , \quad \tau_i^{(1)} = \tau_i^{(1a)} + \tau_i^{(1b)}\ . \qquad (27)$$

General results for the longitudinal parts of the vertex are reasonably compact, even in a general covariant gauge and arbitrary dimension (see in [10]). Below, we show a few selected results.

3.1. Longitudinal part

The λ functions for diagram a are

$$\lambda_1^{(1a)}(p_1^2,p_2^2,p_3^2)=\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\frac{(n-2)(1-\xi)}{4p_1^2p_2^2}$$
$$\times\left[p_1^2(p_2^2{+}m^2)\kappa_{1,2}+p_2^2(p_1^2{+}m^2)\kappa_{1,1}-(p_1^2{+}p_2^2)m^2\widetilde{\kappa}\right],$$
$$\lambda_2^{(1a)}(p_1^2,p_2^2,p_3^2)=\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\frac{(n-2)(\xi-1)}{4p_1^2p_2^2(p_1^2-p_2^2)}$$
$$\times\left[p_1^2(p_2^2{+}m^2)\kappa_{1,2}-p_2^2(p_1^2{+}m^2)\kappa_{1,1}-(p_1^2{-}p_2^2)m^2\widetilde{\kappa}\right],$$
$$\lambda_3^{(1a)}(p_1^2,p_2^2,p_3^2)=-\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\frac{(n-\xi)m}{p_1^2-p_2^2}$$
$$\times[\kappa_{1,2}-\kappa_{1,1}],$$
$$\lambda_4^{(1a)}(p_1^2,p_2^2,p_3^2)=0. \tag{28}$$

For diagram b, the results are a bit longer. Here we present the λ's in Feynman gauge:

$$\lambda_1^{(1b)}(p_1^2,p_2^2,p_3^2)\Big|_{\xi=0}=-\frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{1}{8}\Bigg[2(p_1^2{+}p_2^2{-}2m^2)\varphi_1$$
$$-4\kappa_{0,3}-n\kappa_{1,2}-n\kappa_{1,1}$$
$$+\frac{(p_1^2-p_2^2)}{\mathcal{K}}\Big((p_1^2-p_2^2)\{[(p_1p_2)+m^2]\varphi_1+\kappa_{0,3}\}$$
$$+[p_2^2-(p_1p_2)]\kappa_{1,2}-[p_1^2-(p_1p_2)]\kappa_{1,1}\Big)$$
$$-(n-2)\left(\frac{m^2}{p_2^2}\kappa_{1,2}+\frac{m^2}{p_1^2}\kappa_{1,1}-\frac{p_1^2+p_2^2}{p_1^2p_2^2}m^2\widetilde{\kappa}\right)\Bigg],$$
$$\lambda_2^{(1b)}(p_1^2,p_2^2,p_3^2)\Big|_{\xi=0}=\frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{1}{8}\Bigg[\frac{p_3^2}{\mathcal{K}}\Big([(p_1p_2){+}m^2]\varphi_1$$
$$+\kappa_{0,3}+\frac{p_2^2-(p_1p_2)}{p_1^2-p_2^2}\kappa_{1,2}-\frac{p_1^2-(p_1p_2)}{p_1^2-p_2^2}\kappa_{1,1}\Big)$$
$$+(n-2)\left(\frac{p_1^2{+}m^2}{p_1^2(p_1^2{-}p_2^2)}\kappa_{1,1}-\frac{p_2^2{+}m^2}{p_2^2(p_1^2{-}p_2^2)}\kappa_{1,2}+\frac{m^2}{p_1^2p_2^2}\widetilde{\kappa}\right)\Bigg],$$
$$\lambda_3^{(1b)}(p_1^2,p_2^2,p_3^2)\Big|_{\xi=0}=\frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{(n-1)m}{2(p_1^2{-}p_2^2)}(\kappa_{1,1}{-}\kappa_{1,2}),$$
$$\lambda_4^{(1b)}(p_1^2,p_2^2,p_3^2)\Big|_{\xi=0}=0. \tag{29}$$

In fact, in an arbitrary gauge ($\xi\neq 0$), $\lambda_4^{(1b)}$ does not vanish. It is proportional to $m\xi$.

3.2. Transverse part

In an arbitrary gauge, the τ's are not so compact as λ's. As an example, we show the result for one of them in the Feynman gauge

$$\tau_1^{(1b)}(p_1^2,p_2^2,p_3^2)\Big|_{\xi=0}=-\frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{(n-1)m}{4\mathcal{K}}$$
$$\times\Bigg(2[(p_1p_2)+m^2]\varphi_1-\kappa_{1,2}-\kappa_{1,1}+2\kappa_{0,3}$$
$$-(p_1-p_2)^2\,\frac{\kappa_{1,1}-\kappa_{1,2}}{p_1^2-p_2^2}\Bigg). \tag{30}$$

Results for all other functions are listed in [10].

In the limit of massless quarks, $m\to 0$, the results simplify considerably. First of all, in this limit, $\tau_1^{(1)}=\tau_4^{(1)}=\tau_5^{(1)}=\tau_7^{(1)}=0$ (this holds also in an arbitrary covariant gauge). Furthermore, $\varphi_1\to\varphi$, $\varphi_2\to\varphi$, $\kappa_{1,i}\to\kappa_{0,i}$, $\kappa_{2,i}\to\kappa_{0,i}$ and $\widetilde{\kappa}\to 0$.

3.3. Renormalization

In the limit $n\to 4$ ($\varepsilon\to 0$) the only function in the quark-gluon vertex which has an ultraviolet (UV) singularity *at one loop* is the $\lambda_1^{(1)}$ function. Absence of UV-singularities in all other functions $\lambda_i^{(1)}$ ($i>1$) and $\tau_i^{(1)}$ was one of the checks on our results. In an arbitrary covariant gauge, the UV-singular part of $\lambda_1^{(1)}$ reads

$$\frac{g^2\eta}{(4\pi)^{2-\varepsilon}}\left[(1-\xi)C_F+\frac{1}{4}(4-\xi)C_A\right]\left(\frac{1}{\varepsilon}+\ldots\right).$$

To renormalize the results, we need to subtract the $1/\varepsilon$'s, (possibly) getting some constant R instead, $(1/\varepsilon+\ldots)\to(R+\ldots)$, which depends on the renormalization scheme. In the $\overline{\text{MS}}$ scheme $R=0$, because (see Eq. (20)) $\eta=e^{-\gamma\varepsilon}\left[1+\mathcal{O}(\varepsilon^2)\right]$, so that $e^{-\gamma\varepsilon}$ and $(4\pi)^\varepsilon$ are absorbed by the $\overline{\text{MS}}$ re-definition of the coupling constant g^2.

4. SPECIAL LIMITS

A few limits are of special interest: (i) the on-shell limit $p_1^2=p_2^2=m^2$ (with or without assuming that the vertex function is being sandwiched between Dirac spinors); (ii) the zero-momentum limit, when the gluon momentum vanishes ($p_3=0$); (iii) the symmetric case, when $p_1^2=p_2^2=p_3^2$. Since in all these cases we can put $p_1^2=p_2^2\equiv p^2$, we start by considering this as the first step towards all these limits.

In the case $p_1^2=p_2^2\equiv p^2$ some of the tensor structures in the quark-gluon vertex become linearly dependent, namely: $L_{2,\mu}$ and $T_{2,\mu}$, $L_{3,\mu}$ and $T_{1,\mu}$, $T_{4,\mu}$ and $T_{7,\mu}$. Moreover, according to Eq. (16), λ_4 and τ_6 vanish. Therefore, the quark-gluon vertex in this limit can be written as [cf.

Eqs. (14) and (13)]

$$\Gamma_\mu|_{p_1^2=p_2^2\equiv p^2} = L_{1,\mu}\lambda_1 + L_{2,\mu}\widetilde{\lambda}_2 + L_{3,\mu}\widetilde{\lambda}_3 + T_{3,\mu}\tau_3 + T_{5,\mu}\tau_5 + T_{7,\mu}\widetilde{\tau}_7 + T_{8,\mu}\tau_8. \quad (31)$$

Only the $L_{1,\mu}$ contribution remains non-transverse in this limit.

4.1. On-shell limit

The tensor structure of (31) does not change when we put $p^2 = m^2$, without assuming that the vertex is sandwiched between Dirac spinors. If we keep n as an arbitrary parameter, the limit $p^2 \to m^2$ is regular for all scalar functions involved in Eq. (31). For example,

$$\lambda_1^{(1)}(m^2, m^2, p_3^2) = \tfrac{1}{4}\frac{g^2\eta}{(4\pi)^{n/2}}\left\{(2-\xi)C_A\kappa_{0,3} + \frac{n-2}{n-3}\left[2(1-\xi)C_F + C_A\right]\widetilde{\kappa}\right\}. \quad (32)$$

An interesting feature is that in this limit the integral $J_2(1,1,1)$ reduces to the two-point function (with a factor of $(n-4)^{-1}$ in front), whereas $J_1(1,1,1)$ remains non-trivial.

If we recall that the "physical" quark-gluon vertex should be sandwiched between physical states obeying the Dirac equation, then we arrive at

$$\overline{u}(-p_1)\Gamma_\mu u(p_2) = F_1(p_3^2)\,\overline{u}(-p_1)\gamma_\mu u(p_2) - \tfrac{1}{2m}F_2(p_3^2)\,\overline{u}(-p_1)\sigma_{\mu\nu}\,p_3^\nu u(p_2), \quad (33)$$

where $F_1(p_3^2)$ and $F_2(p_3^2)$ are often called the Dirac and Pauli form factors, respectively. In terms of the (modified) longitudinal and transverse functions we get

$$\begin{aligned} F_1 + F_2 &= \lambda_1 + p_3^2\tau_3 - 2m\tau_5 + \tfrac{1}{2}(p_3^2 - 4m^2)\tau_8, \\ \tfrac{1}{2m}F_2 &= -2m\widetilde{\lambda}_2 + \widetilde{\lambda}_3 - \tau_5 + \tfrac{1}{2}p_3^2\widetilde{\tau}_7 - m\tau_8. \quad (34) \end{aligned}$$

4.2. Zero-momentum limit

Let us now consider the off-shell zero-momentum limit $p_3 = 0$ $(p_2 = -p_1 \equiv p)$. Upon putting $p_1^2 = p_2^2 = p^2$, the next step is to put $p_3^2 = 0$ (which implies $p_3 = 0$). In this limit, most of the transverse structures vanish and the quark-gluon vertex looks like

$$\begin{aligned} \Gamma_\mu|_{p_2=-p_1\equiv p,\ p_3=0} &= L_{1,\mu}\lambda_1 + L_{2,\mu}\widetilde{\lambda}_2 + L_{3,\mu}\widetilde{\lambda}_3 \\ &= \gamma_\mu\lambda_1 + 4p_\mu \not{p}\widetilde{\lambda}_2 - 2p_\mu\widetilde{\lambda}_3. \quad (35) \end{aligned}$$

Of course, all integrals can be reduced to two-point functions and tadpoles. The corresponding scalar functions are regular in this limit.

In the massless case ($m = 0$), only two relevant structures remain in (35). The corresponding results have been compared with [8].

4.3. Symmetric case

The limit $p_1^2 = p_2^2 = p_3^2 \equiv p^2$ is interesting for studying the Z-factors and renormalization group quantities in the MOM scheme. Usually, a Euclidean symmetric point is considered, $p^2 = -\mu^2$. For massless quarks, the one-loop calculation of the quark-gluon vertex was performed in [5] (see also in [6]). For massive quarks, one of the scalar functions, the coefficient of γ_μ, was presented in [7]. We confirm all these results, and also have obtained expressions for the remaining functions (for details, see in [10]).

5. CONCLUSIONS

We have reviewed results for the one-loop quark-gluon vertex in an arbitrary covariant gauge and space-time dimension. The calculation was carried out with massive quarks.

We decomposed the quark-gluon vertex into longitudinal and transverse parts (6) (like the decomposition in QED [2]). In general, 12 scalar functions (four λ's and eight τ's) are needed to define the quark-gluon vertex. In particular, we have found that the function λ_4, the coefficient of $\sigma_{\mu\nu}(p_1 - p_2)^\nu$, which does not appear in QED (at least, at the one-loop level), does not vanish in QCD. Moreover, it contributes to the non-Abelian sector of the WST identity (3). Various special cases of the general results were compared with those of Refs. [2–7,22] (see in [10]).

Starting from the general off-shell expressions in an arbitrary space-time dimension, n, for the longitudinal and transverse parts of the vertex, we have derived results for the on-shell limits which also are valid for an arbitrary value of the space-time dimension. For special cases, our on-shell results have been compared against those from Refs. [8,9,23,24].

We have calculated other functions involved and checked that our results obey the WST identity (3), for arbitrary n and ξ.

In principle, some techniques which can be used

for the calculation of the two-loop off-shell quark-gluon vertex, at least in the $m = 0$ case, are already available [25,17], although the problem of higher powers of irreducible numerators is still difficult for algorithmization. For special limits, the calculation is very similar to the three-gluon vertex, which was calculated at two loops in [26] (the zero-momentum limit) and in [27] (the on-shell case).

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